Stochastic Modeling of Carrier Sensing Based Cognitive Radio Networks

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Abstract

In this paper, we propose a comprehensive probabilistic framework which can be used to model and analyze cognitive radio (CR) network using carrier sensing (CS) based multiple access scheme. We then discuss several CR network models as case studies. For each model, analytical results are derived for important performance metrics. This leads to a quantification of the interplay between primary and secondary users in such networks.

1. Introduction

Nowadays, the rapid development of wireless communication is facing a paradox- the 'un-real' bandwidth scarcity. The unlicensed band is becoming too crowded while in the licensed band, measurements [1] show that most of the spectrum is vastly underutilized. This manifests itself through voids in either time, space or spectrum. Dynamic spectrum access aims at exploiting these voids to accommodate extra radio devices (referred to as secondary users) in a network using some licensed band, whose first function is to serve a population of licensed users (referred to as primary users).

Cognitive radios [2],[21], or programmable radios are radio devices which are capable to detect and adapt their transmissions to various network environments. These radio devices are those to used by the secondary users. The essential difference between a network featuring both primary and secondary users, also known as cognitive radio (CR) network, and a normal network can be mainly characterized by the interplay between the two classes of users. In a CR network, the primary users selfishly protect their own transmissions while the secondary users must adapt their transmissions to guarantee the privilege of the former as much as possible.

There are many essential questions regarding how we can successfully implement CR: How can we effectively and accurately detect the spectrum voids? These voids are in time, space or frequency. What is the best mechanism to enforce secondary users to respect the preeminence of primary users? How can we quantify the service to secondary users as well as the preeminence of primary users?... One might also ask about the implementation of CR at physical layer, MAC layer... There is a very rich literature aiming at answering these questions. Spectrum detection is one of the core problems in cognitive radio and is addressed in [8],[15],[16] while the papers [17],[4],[18] aim at solving the network (frequency) selection problem. Some power control schemes are proposed in [13],[14]. In [9],[10],[11],[12] implementations of CR exploiting voids in time and frequency are considered, while [20] aims at exploiting voids in space. Some protocols for CR are also proposed for physical layer as well as MAC layer in [9],[11],[12]. Even if some first models were proposed in [3],[19], it is fair to say that there is a strong need for a comprehensible mathematical framework for the modeling and analysis of such CR networks. The main object of the present paper is a new probabilistic framework based on stochastic geometry [6], [7] for the modeling and analysis of a wide range of MAC protocols within this context.

More specificly, in this paper, we focus on a class of CR MAC protocols exploiting voids in *space* for secondary transmissions and using carrier sensing (CS) to detect spectrum voids. In wireless communication, CS is often used to avoid collisions arising when users which are too close transmit at the same time. In the CR context, CS can be used to avoid collisions arising when a secondary user which is too close to a transmitting primary user transmits at the same time. This protects the primary users. The interplay between the two classes of users in a CR network using a CS based MAC protocol mimics that of priority queues and can be expressed in simple geometric terms as follow: each primary transmitter demands a protection zone. If an other user located in this zone transmits at the same time, a collision occurs. Any user located in this protection zone is hence called a *contender* of this transmitter. In most cases, the union of all primary transmitters protection zones does not cover the whole space. Thus, one can accommodate secondary users in the remaining space to better utilize the spatial resource.

The method introduced in this paper was first developed in [20] to analyze a CSMA like CR network, which will be referred to as the Cognitive-CSMA. In this type of networks, CS is used not only to protect primary transmissions from secondary interferer but also to control interference within each class. The present paper will bring a broader view upon this approach where the stochastic framework will be extended to cope with more general contexts.

Generally, in CR networks, a secondary user uses CS to guarantee that it does not belong to any primary user protection zone. Given that this condition is satisfied, any MAC protocol can be employed within each class. We will consider here three examples such networks. We will first consider a single primary user network and a broadcast primary users network; then we will revisit the Cognitive-CSMA network considered in [20]. These network models are not only just toy examples: the single primary user model can be used as the basic building block of any CR network; the broadcast primary user one can be used to model CR TV networks or cellular networks whereas the Cognitive-CSMA setting gives us a fully distributed MAC protocol for CR mobile ad hoc networks. The analysis of these networks can give us much insight on the interaction between the two classes of users and help us establish guarantees for primary users.

The potential of this stochastic framework nevertheless does not stop here. One could e.g. consider within this framework CR networks using TDMA to schedule primary transmissions and Aloha to schedule secondary transmissions; one could also use CSMA for primary users and tree based protocols for secondary ones or vice versa, etc. Each of these networks has its own beautiful geometric properties and realistic implications and the fact that we only consider the 3 examples listed above is merely due to space constraints.

The remainder of this paper is structured as follow: In sections 2, 3 and 4 we give the stochastic frameworks and the main results for our three CR network model examples. Section 5 contains our conclusion.

2. Single primary user

In this section we consider a CR network model with a single primary user and a population of secondary users. This models a CR network where the primary population is so sparse that the intra-class interference is negligible. The presence of secondary users is nevertheless still causing interference to the primary user due to the pervasive nature of radio communication. The goal of this section is to quantify this 'secondary to primary' interference and its impact on the performance of the primary user.

In this model, there is only one primary user and the secondary users employ an ALOHA based MAC protocol within their class. Each secondary user independently tosses a coin. If the result is head, then this user will sense the network to see if there is any primary contender. The tagged secondary user transmits only if it sees the network free of primary contender. The bias of these coins is a pre-set parameter of the protocol.

2.1. User Model

This model features a primary user which is a transmitter-receiver pair T_0^I, R_0^I and a population of secondary users (transmitter-receiver pairs) $\{T_i^{II}, R_i^{II}\}$. By abuse of notation we will use also this notation for the positions of transmitters and receivers. We assume:

- T_0^I is at the center of the plane.
- $|R_0^I| = R$.
- θ_0^I is the angular position of R_0^I.
- The process of transmitters who access to the network is given by ALOHA (the coin is head) $\Phi^{II} = \{T_i^{II}\}$ is assumed to form a realization of a homogeneous Poisson point process (p.p.p.) of intensity λ^{II}.
- Each secondary receiver is assumed to be uniformly distributed on a circle of radius r centered at its transmitter.
- $R_i^{II} = T_i^{II} + rl(\theta_i^{II})$ where θ_i^{II} is uniformly, identically and independently distributed (i.i.d) in $[0, 2\pi]$, and $l(\theta) = (cos(\theta), sin(\theta))$.
- $F_{0,i}^{I-II}$ ($F_{i,0}^{II-I}$, $F_{i,j}^{II-II}$, $F_{0,0}^{I-I}$ resp.) is the fading of the channel from T_0^I to R_i^{II} (T_i^{II} to R_0^{II}, T_i^{II} to T_j^{II}, T_0^I to R_0^I resp.). All the fading variables are assumed to be i.i.d. and to follow the cumulative distribution function (C.D.F) $G(.) = \mathbb{P}(F < .)$. In this paper we assume Rayleigh fading, i.e. $G(x) = 1 - e^{-\mu x}$. For more about fading, the reader should refer to [22].

2.2. Retain and transmission model

Throughout the paper we will assume a deterministic fading. We can then define the indicator variable that secondary user i belongs to the primary user's protection zone as

$$U_i = \mathbf{1}_{F_{i,0}^{II-I}|T_i^{II} - R_0^I|^{-\alpha} > \rho},$$

where ρ is a pre-set constant.
Interference is treated as noise and the transmission is successful if the signal to noise and interference ratio (SINR) is larger than a pre-set constant T. The SINRs of the primary user and secondary users are defined as:

$$SINR_0^I = \frac{F_{0,0}^{I-I}/R^\alpha}{W(R_0^I)+I_{\Phi_M^{II}}(R_0^I)}$$

$$SINR_0^{II} = \frac{F_{i,i}^{II-II}/r^\alpha}{W(R_i^{II})+F_{0,i}^{I-II}/|R_i^{II}|^\alpha+I_{\Phi^{II}\setminus T_i^{II}}(R_i^{II})},$$

where W is the thermal noise, $\Phi_M^{II} = \{T_j^{II} \text{ s.t } U_j = 1\}$ and $I_\Xi(x)$ is the Shot-Noise associated with the point process Ξ at point x, defined by:

$$I_\Xi(x) = \sum_{X_i \in \Xi} f(|X_i - x|)$$

with f some response function. For more about Shot-Noise processes and Stochastic geometry, the reader should refer to [7].
Particular cases of interest are:

$$I_{\Phi_M^{II}}(R_0^I) = \sum U^j F_{j,0}^{II-I}/|T_j^{II}-R_0^I|^\alpha$$

$$I_{\Phi^{II}\setminus T_i^{II}}(R_i^{II}) = \sum_{j\neq i} U^j F_{j,i}^{II-I}/|T_j^{II}-R_i^{II}|^\alpha.$$

2.3. Performance analysis

In this subsection we investigate several important performance metrics, namely the *medium access probability* (MAP), the *coverage probability* (COP) and the *total throughput* (TT). The MAP is defined as the probability that a user accesses the channel. In this model the primary user always has access to the channel, thus we are only interested in the MAP of a secondary user which is $\mathbb{P}(U_i = 1)$. The second metric of interest is the COP. Due to interference, not every transmission attempt is successful. The COP measures this probability of success and is computed as the probability that the SINR of the tagged user is larger than a pre-set constant T. The COP nevertheless only measures the network performance at each user; if one is interested in the global network performance, then one should consider the TT. This is defined as the mean number of successful transmissions in the network per time slot. This tells us how fast the network functions at a global scale.
For the model discussed here, the above metrics can be computed as in the following propositions:

Proposition 1 *Conditioned on the positions of transmitters and receivers, the MAP of the i^{th} secondary user is:*

$$1-\exp\{-\mu\rho|T_i^{II}-R_0^I|^\alpha\}. \tag{1}$$

Proposition 2 *Assume Rayleigh fading. Conditioned on the primary receiver position, the COP of the primary user is:*

$$p_c^I = \mathscr{L}_W(\mu T R^\alpha)\exp\{-\lambda^{II}\Psi\}, \tag{2}$$

with

$$\Psi = \int_{\mathbb{R}^2} g(x,R_0^I)\frac{\int_0^{2\pi}(1-e^{-\mu\rho|x+rl(\theta)|^\alpha})d\theta}{2\pi} \tag{3}$$

$$g(x,y) = 1-e^{-\mu\rho|x-y|^\alpha} - \frac{|x-y|^\alpha e^{-\mu\rho(TR^\alpha+|x-y|^\alpha)}}{TR^\alpha+|x-y|^\alpha}. \tag{4}$$

Moreover, this COP is independent of R_i^I.

Proposition 3 *Assume Rayleigh fading. Conditioned on the secondary transmitter position y and the primary receiver position, the COP of that primary user is:*

$$p_c^{II}(y,\lambda^{II}) = \frac{\mathscr{L}_W(\mu Tr^\alpha)}{2\pi}\int_0^{2\pi}\frac{|y+rl(\theta)|^\alpha}{|y+rl(\theta)|^\alpha+Tr^\alpha} e^{-\lambda^{II}\int_{\mathbb{R}^2}\frac{Tr^\alpha(1-e^{-\mu\rho|x-R_0^I|^\alpha})}{Tr^\alpha+|x-y-rl(\theta)|^\alpha}dx}d\theta. \tag{5}$$

It is easy to see that the total throughput of all the secondary users on the whole plane is infinite, thus we consider only the secondary users in a region C which is a disk of radius $R_{max} \gg \max\{R,r\}$ centered at T_0^I. The intuition behind this choice of C is that, for secondary users outside C, the interactions with the primary users is negligible. In fact their MAPs are almost 1 and the interference from the primary user to them is negligible. Thus those secondary users behave as users in a normal ALOHA network.

Proposition 4 *Conditioned on the primary receiver position, the TT of secondary users within C is:*

$$S^{II}(\lambda^{II}) = \lambda^{II}\int_C p_c^{II}(y,\lambda^{II})(1-e^{-\mu\rho|y-R_0^I|^\alpha})dy. \tag{6}$$

Moreover, this TT is independent of R_i^I.

2.4. CR guarantees

In this subsection we discuss the policies that secondary users have to comply with in order to provide some guarantees to the primary user. From (2) we can see that $p_c^I(\lambda^{II})$ decreases exponentially fast to 0 as λ^{II} goes to ∞. Thus, for $1 > L > 0$, there exists a unique λ^{up} such that $p_c^I(\lambda^{up}) = L$. If one wishes to have a stochastic guarantee that the COP of the primary user is at least L, then one ought to limit the density of secondary users below λ^{up}. Within this constrain, the secondary users seek to optimize their TT, i.e. maximizing

$S^{II}(\lambda^{II})$. Let $\lambda^{max} = \arg\max\{S^{II}(.)\}$; the optimal operation point in this context consists in setting secondary intensity equal to $\lambda^* = \min\{\lambda^{up}, \lambda^{max}\}$. This can be done in a distributed way by requiring each secondary user to adjust its ALOHA coin tossing bias such that the intensity of secondary user accessing the network is λ^*.

3. Multicast primary user

In this section we investigate a CR network model featuring a multicast primary user, i.e. a primary transmitter with a population of primary receivers, and a population of secondary users. For example, the primary transmitter can be a TV station (in a TV network) or a base station (in a cellular system). As in the above model, the other base stations are assumed to be so far that the inter-cell interference is negligible. Thus, the only factor that has negative impact on the primary user performance is the 'secondary to primary' interference. In this model, each primary receiver requires a protection zone around it and secondary users must use CS to guarantee that they do not belong to any of these protection zones.

3.1. User model

As the previous model, we use the same notation for nodes and position of nodes. We assume:

- The primary transmitter T^I_0 is at the center of the plane.
- The process of primary receivers $\{R^I_i\}$ forms a realization of a p.p.p. of intensity $\lambda^I \mathbf{1}_C$, where C is the cell of the base station. In this paper we assume that C is a disk of radius R centered at T^I_0.
- The process of secondary transmitters $\Phi^{II} = \{T^{II}_i\}$ is assumed to form a realization of a p.p.p. of intensity $\lambda^{II}\mathbf{1}_C$. Thus any secondary user outside C belongs to other cells and is not considered.
- The secondary receiver R^{II}_i is assumed to be uniformly distributed on the circle of radius r centered at T^{II}_i, i.e. $R^{II}_i = T^{II}_i + rl(\theta^{II}_i)$ where $l(.)$ is defined is the previous section and θ^{II}_i is i.i.d. in $[0, 2\pi]$.
- $F^{I-I}_{0,i}$ ($F^{I-II}_{0,i}$,$F^{II-I}_{i,j}$,$F^{II-II}_{i,j}$ resp.) is the fading of the channel from T^I_0 ro R^I_i (T^I_0 to R^{II}_i, T^{II}_i to R^I_j, T^{II}_i to R^{II}_j). The fading variables are i.i.d and follow the same C.D.F which is assumed to be $G(x) = 1-e^{-\mu x}$ (Rayleigh fading).

3.2. Retain and transmission model

In this context, a secondary user is allowed to transmit if it does not cause too much interference to any of the primary receivers. Namely, the retain indicator of the i^{th} secondary user is:

$$U_i = \prod_j \mathbf{1}_{F^{II-I}_{i,j}/|T^{II}_i - R^I_j|^\alpha < \rho},$$

where ρ is a pre-set constant. As usual, the transmission scheme treats interference as noise and a transmission is successful if the SINR is higher than a threshold T. The SINR is defined for primary and secondary receivers as:

$$SINR^I_i = \frac{F^{I-I}_{0,i}/|R^I_i|^\alpha}{W(R^I_i) + I_{\Phi^M}(R^I_i)}$$
$$SINR^{II}_i = \frac{F^{II-II}_{i,i}/r^\alpha}{W(R^{II}_i) + F^{I-II}_{0,i}/|R^{II}_i|^\alpha + I_{\Phi^M\setminus T^{II}_i}(R^{II}_i)}.$$

In the above formulas, $\Phi^M = \{T^{II}_i \text{ s.t. } U_i = 1\}$ is the process of retained secondary transmitters and the Shot-noise interference is defined as:

$$I_{\Phi^M}(R^I_i) = \sum_j U_j F^{II-I}_{j,i}/|T^{II}_j - R^I_i|^\alpha$$
$$I_{\Phi^M\setminus T^{II}_i}(R^{II}_i) = \sum_{j\neq i} U_j F^{II-II}_{j,i}/|T^{II}_j - R^{II}_i|^\alpha.$$

3.3. Performance analysis

In this subsection, the performance metrics of interest are still the MAP, COP and TT, for which definitions are provided in Section 2. We will proceed directly to the main results which quantify these metrics in this context.

Proposition 5 *Conditioned on the position y of a secondary transmitter, the MAP of this secondary user is:*

$$\exp\{-\lambda^I N(y)\}, \tag{7}$$

where:

$$N(y) = \int_C \exp\{\mu\rho|y-x|^\alpha\}dx. \tag{8}$$

Proposition 6 *Conditioned on the position y of a primary receiver, the COP of this receiver is:*

$$p^I_c(y, \lambda^I, \lambda^{II}) = \mathscr{L}_W(\mu T|y|^\alpha)\exp\{-\lambda^{II} \int_C g(x,y)e^{-\lambda^I N(x)}dx\} \tag{9}$$

Proposition 7 *Conditioned on the position y of a secondary transmitter, the COP for this user is:*

$$p_c^{II}(y,\lambda^I,\lambda^{II}) = \frac{\mathscr{L}_W(\mu T r^\alpha)}{2\pi}\int_0^{2\pi}\frac{|y+lr(\theta)|^\alpha}{Tr^\alpha+|y+lr(\theta)|^\alpha}$$
$$\exp\{-\lambda^{II}\int_C\frac{Tr^\alpha}{Tr^\alpha+|x-y-lr(\theta)|^\alpha}e^{-\lambda^I N(x)}dx\}d\theta. \tag{10}$$

Proposition 8 *The TT is*

$$S^I(\lambda^I,\lambda^{II}) = \lambda^I\int_C p_c^I(y,\lambda^I,l\lambda^{II}), \tag{11}$$

for primary users and

$$S^{II}(\lambda^I,\lambda^{II}) = \lambda^{II}\int_C p_c^{II}(y,\lambda^I,\lambda^{II})e^{-\lambda^I N(y)}, \tag{12}$$

for secondary users.

3.4. CR guarantees

As in subsection 2.4, we seek for an operation point that complies with performance guarantees for primary users and at the same time maximizes the performance of secondary users. In this model, instead of considering the local COP of each primary receiver, we consider the global performance metric: the spatial throughput. From (11), we can have two important remarks. First, the TT $S^I(\lambda^I,\lambda^{II})$ of primary users increases almost linearly in the primary receivers intensity λ^I. This comes from 2 reasons: increasing λ^I makes the MAPs of secondary users decrease exponentially fast as shown in (7) and thus decreases inter-class interference; increasing λ^I also increases the number of primary receivers and makes the TT increases almost linearly. The second important remark is that $S^I(\lambda^I,\lambda^{II})$ decreases exponentially fast to 0 as λ^{II} goes to ∞. This means that in spite of the protection zones, inter-class interference from an over-crowded area of secondary users can destroy any primary transmission. Thus, a limitation on the secondary users intensity, which is similar to that in subsection 2.4, ought to be applied to stochastically guarantee an acceptable performance for primary users. More precisely, for $L > 0$, there exists a unique λ^{up} such that $S^I(\lambda^I,\lambda^{up}) = L$. One wants to guarantee a minimum TT L for primary users; thus the secondary users intensity must be limited to be smaller than λ^{up}. Within this constraint, one seeks for an operation point maximizing the spatial throughput $S^{II}(\lambda^I,\lambda^{II})$. Let $\lambda^{max} = \arg\max\{S^{II}(\lambda^I,.)\}$; then the optimal operation point is setting the secondary user intensity equal to $\lambda^* = \min\{\lambda^{up},\lambda^{max}\}$. This optimal operation point can be enforced in an almost distributed way by requiring each secondary user to adjust the ALOHA coin bias similarly to that presented in subsection 2.4. However, this time the scheme is only 'almost' distributed since for the tuning of the coin bias, each secondary user has to know the 'global' parameter λ^I .

4. Cognitive-CSMA

In this section, we consider a CR network with many primary transmitters. This type of CR network is more complex than the networks investigated above in the sense that a primary user has to suffer from both inter-class and intra-class interference. Thus, in addition to the CS process used by secondary users to limit inter-class interference, a CSMA protocol is applied within each class to limit intra-class interference. The resulting protocol is called Cognitive-CSMA. In this CSMA protocol, CS is used to detect contenders, i.e. users who are too close, and a back off timer mechanism is used to guarantee that no two contenders can transmit at the same time. The detail of the back-off timer will be discussed later in the network model.

4.1. User model

In this network model, instead of considering a network with users distributed on a restricted area, we consider a large network with users distributed on the whole plane. The network features a process of primary users and a process of secondary users. Each user is a transmitter-receiver pair as follow:

- The process of primary transmitters $\Phi^I = \{T_i^I\}$ forms a realization of a homogeneous p.p.p. of intensity λ^I .
- The process of primary transmitters: $\Phi^{II} = \{T_i^{II}\}$ forms a realization of a homogeneous p.p.p. of intensity λ^{II}.
- The receivers are assumed to be uniformly distributed on a circle of radius r centered at its receivers:
$$R_i^I = T_i^I + rl(\theta_i^I)$$
$$R_i^{II} = T_i^I + rl(\theta_i^{II})$$
- The back-off timers are t_i^I and t_i^{II} for primary and secondary users resp.
- $F_{i,j}^{I-I}$ ($F_{i,j}^{I-II}$,$F_{i,j}^{II-I}$,$F_{i,j}^{II-II}$,$F_{i,j}^{I-I}$ resp.) are fadings of the channel from T_i^I to R_j^I (T_i^I to R_j^{II},T_i^{II} to R_j^I,T_i^{II} to R_j^{II} resp.). These fading variables are i.i.d and follow the C.D.F $G(x) = 1-\exp\{-\mu x\}$ (Rayleigh fading).

4.2. Retain and transmission model

The details of the back-off timer mechanism are as follows: A primary user is retained if it has the smallest timer among its primary contenders. A secondary user is retained if it has no primary contender and it has the smallest timer among its secondary contenders. These conditions can be implemented by a 2 stage sensing method: A primary and a secondary sensing phase are included in each time slot. During the primary sensing phase, a primary user starts sensing the network and informs other users about its presence when its timer expires; a secondary user constantly senses the network and is refrained if it senses any primary contender. During the secondary sensing phase, a secondary user starts sensing the network and informs other users about its presence when its timer expires. Thus a primary user with the smallest timer among its contender will sense the network first and is retained; any contender sensing the network after him will see him and refrains from transmitting. A secondary user with no primary contender and the smallest timer among its secondary contenders will sense the network first in the secondary sensing phase and is retained; any secondary contender sensing the network after him will see him and refrains from transmitting.
Let

$$\mathcal{N}_{I,i}^{I} = \{j \text{ s.t. } j \neq i \text{ and } F_{i,j}^{I-I} > \rho|T_j^I - R_i^I|^\alpha\} \quad (13)$$

$$\mathcal{N}_{II,i}^{I} = \{j \text{ s.t. } F_{i,j}^{I-II} > \rho|T_j^I - R_i^{II}|^\alpha\} \quad (14)$$

$$\mathcal{N}_{II,i}^{II} = \{j \text{ s.t. } j \neq i \text{ and } F_{i,j}^{II-II} > \rho|T_j^{II} - R_i^{II}|^\alpha\}, \quad (15)$$

be the set of primary contenders of the i^{th} primary user, the set of primary contenders of the i^{th} secondary user, the set of secondary contenders of the i^{th} secondary user, resp. The retain condition can be encoded as follow:

$$U_i^I = \mathbf{1}_{t_i^I < t_j^I \forall j \in \mathcal{N}_{I,i}^I} \quad (16)$$

$$U_i^{II} = \mathbf{1}_{\mathcal{N}_{II,i}^I = \emptyset} \mathbf{1}_{t_i^{II} < t_j^{II} \forall j \in \mathcal{N}_{II,i}^{II}}. \quad (17)$$

As in the previous models, we treat interference as noise so that a transmission is successful if the SINR is higher than the threshold T. Using the Shot-noise interference notation we can write SINR for primary and secondary users as:

$$SINR_i^I = \frac{F_{i,i}^{I-I}/r^\alpha}{W(R_i^I) + I_{\Phi_M^I \setminus T_i^I}(R_i^I) + I_{\Phi_M^{II}}(R_i^I)}$$

$$SINR_i^{II} = \frac{F_{i,i}^{II-II}/r^\alpha}{W(R_i^{II}) + I_{\Phi_M^I}(R_i^{II}) + I_{\Phi_M^{II} \setminus T_i^{II}}(R_i^{II})}.$$

As above, W is thermal noise. The Shot-noise interference can be defined as follow:

$$I_{\Phi_M^I \setminus T_i^I}(R_i^I) = \sum_{j \neq i} U_i^I F_{j,i}^{I-I} / |T_j^I - R_i^I|^\alpha$$

$$I_{\Phi_M^{II}}(R_i^I) = \sum_j U_j^{II} F_{j,i}^{II-I} / |T_j^{II} - R_i^I|^\alpha$$

$$I_{\Phi_M^{II} \setminus T_i^{II}}(R_i^{II}) = \sum_{j \neq i} U_i^{II} F_{j,i}^{II-II} / |T_j^{II} - R_i^{II}|^\alpha$$

$$I_{\Phi_M^I}(R_i^{II}) = \sum_j U_j^I F_{j,i}^{I-II} / |T_j^I - R_i^{II}|^\alpha.$$

4.3. Performance analysis

For this CR network, the local performance metrics are again the MAP and the COP. However, for the global performance metric, instead of TT we consider the *spatial density of throughput* (SDT) since the former is infinite in this context as we will see latter. The SDT is defined as the average number of successful transmissions in the network per time slot per unit of surface. Let us first begin with the MAP.
Unlike the previous models, even the primary users have to contend with each other, thus we have to determine the MAP for both primary and secondary users:

Proposition 9 *For Cognitive-CSMA the MAPs of typical primary and secondary users are:*

$$p_I(\lambda, \boldsymbol{p}) = \frac{1 - e^{-\lambda^I \overline{N}_0^r}}{\lambda^I \overline{N}_0^r} \quad (18)$$

$$p_{II}(\lambda, \boldsymbol{p}) = \frac{1 - e^{-\lambda^{II} \overline{N}_0^r}}{\lambda^{II} \overline{N}_0^r} e^{-\lambda^I \overline{N}_0^r}. \quad (19)$$

with

$$\overline{N}_0^r = \int_{\mathbb{R}^2} \left(1 - \left(\frac{\int_0^{2\pi} G(\rho|x - rl(\theta)|^\alpha) d\theta}{2\pi}\right)^2\right) dx. \quad (20)$$

For the COP we have to change a little bit the analysis method. By introducing the virtual timer:

$$\tau_i^I = \frac{\lambda^I}{\lambda^I + \lambda^{II}} t_i^I \qquad \tau_i^{II} = \frac{\lambda^I}{\lambda^I + \lambda^{II}} + \frac{\lambda^{II}}{\lambda^I + \lambda^{II}} t_i^{II},$$

the CR network model considered here is converted to the model of a CSMA network considered in [6] with user intensity $\lambda^I + \lambda^{II}$ and the primary users play the role of the users having a timer smaller than $\frac{\lambda^I}{\lambda^I + \lambda^{II}}$. Since we now consider this as a network with only one class of users, we will use uniformly the notation T_i and R_i for transmitter and receiver positions. Let $\lambda = \lambda^I + \lambda^{II}$, we then obtain the following results:

Proposition 10 *Assume Rayleigh fading. Conditionally on the fact that the network has 2 users i and j such that $R_i - T_j = x$, and the fact that user i is retained by the Cognitive-CSMA/RTS-CTS protocol and conditionally on θ_i, the probability that the protocol also retains j is*

$$g(x,\lambda,\theta_i) = \int_0^{2\pi} \frac{\frac{1}{\lambda(c(x,\theta_i,\theta)-\overline{N}_0^r)}\left(\frac{1-e^{-\lambda\overline{N}_0^r}}{\lambda\overline{N}_0^r} - \frac{1-e^{-\lambda c(x,\theta_i,\theta)}}{\lambda c(x,\theta_i,\theta)}\right)k(x,\theta_i,\theta)}{\pi\left(\frac{1-e^{-\lambda\overline{N}_0^r}}{\lambda\overline{N}_0^r} - (1-k(x,\theta_i,\theta))\left(\frac{1-e^{-\lambda\overline{N}_0^r}}{(\lambda\overline{N}_0^r)^2} - \frac{e^{-\lambda\overline{N}_0^r}}{\lambda\overline{N}_0^r} - \right)\right)} d\theta, \tag{21}$$

where:

$$c(x,\theta_i,\theta) = 2\overline{N}_0^r - \int_{\mathbb{R}^2} \frac{\int_0^{2\pi}(1-k(z,\theta',\theta_i))(1-k(z-x,\theta',\theta))d\theta'}{2\pi} dz \tag{22}$$

$$k(x,\theta_i,\theta) = (1-e^{-\gamma\rho|x|^\alpha})(1-e^{-\gamma\rho|x-rl(\theta_i)-rl(\theta)|^\alpha}). \tag{23}$$

Proposition 11 *Assume Rayleigh fading. Conditionally on θ_0, when approximating the process of retained transmitters viewed by r_0, the receiver of T_0, by an inhomogeneous Poisson p.p. of intensity measure $\lambda g(x,\lambda,\theta_0)dx$ and with marks as indicated above, the COP of T_0 is:*

$$q_c(\lambda,\theta_0) = \boldsymbol{P}(SINR_0 > T) = \mathscr{L}_W(\gamma T R^\alpha) \, e^{-\lambda \int_{\mathbb{R}^2}\left(1-\frac{|x|^\alpha(1-e^{-\gamma\rho(Tr^\alpha+|x|^\alpha)})}{(Tr^\alpha+|x|^\alpha)(1-e^{-\gamma\rho|x|^\alpha})}\right)g(x,\theta_0)dx}. \tag{24}$$

Moreover $q_c^r(\lambda,\theta_0)$ does not depend on θ_0.

Proposition 12 *The SDT of primary and secondary users are:*

$$S^I(\lambda^I,\lambda^{II}) = \lambda^I p_I(\lambda,\boldsymbol{p})q_c(\lambda) \tag{25}$$
$$S^{II}(\lambda^I,\lambda^{II}) = \lambda^{II} p_{II}(\lambda,\boldsymbol{p})q_c(\lambda). \tag{26}$$

Recall that $\lambda = \lambda^I + \lambda^{II}$.

4.4. CR guarantees

The CR guarantee for Cognitive-CSMA consists in guaranteeing a minimum SDT S for primary users. Note that once the primary users intensity is fixed, this is equal to a stochastic guarantee that the COP of a typical primary user is at least S/λ^I. First, by taking derivative of the exponent in the right hand side of (24) with respect to λ, we have that $q_c(\lambda)$ is decreasing in λ. Second, since

$$\lim_{\lambda\to\infty} q(\lambda) = q = \exp\left(\int_{\mathbb{R}^2}\int_0^{2\pi} \frac{k(x,\theta_i,\theta)}{\pi c(x,\theta_i,\theta)(1+\frac{|x|^\alpha}{TR^\alpha})} d\theta dx\right), \tag{27}$$

there however exists an intrinsic limit on the degradation of the COP of primary users due to inter-class interference, regardless of the intensity of the secondary users. Thus, if $S/\lambda^I < q$, then there should be no constraint on the intensity of secondary users. If $S/\lambda^I > q$, then the intensity of secondary users must be smaller than λ^{up} where λ^{up} is the unique value such that $q_c(\lambda^I + \lambda^{up}) = S/\lambda^I$.
Keeping these constraints in mind, we seek for an operation point that maximizes the SDT of secondary users. For the case $S/\lambda^I < q$, this operation point consists in setting the intensity of secondary users equal to λ^{max}, where $\lambda^{max} = \arg\max\{S^{II}(\lambda^I,.)\}$. For the case $S/\lambda^I > q$, this operation point consists in setting the intensity of secondary users equal to $\lambda^* = \min\{\lambda^{max},\lambda^{up}\}$. This optimal operation point can be implemented by the same scheme as in subsection 2.4.

5. Conclusion

The present paper provides a comprehensive probabilistic framework based on stochastic geometry for modeling CR networks. Using this framework, we can derive analytical results for several CR networks which use carrier sensing as a mean to protect the transmissions of primary users. Based on analytical results we can tune these networks to allow secondary users to efficiently exploit unused spectrum while still complying with the predefined requirements on the acceptable impact on primary users.

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